3D Modelling

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Contents

1 Mathematical foundations

Here are the mathematical foundations that are used in my software. You should be familiar with multiplying matrices with vectors or other matrices.

1.1 Rotation matrices in linear algebra

In linear algebra, we can rotate points or vectors around the x, y and z-axis. The matrices for this rotation are given as[1]:

$$R_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

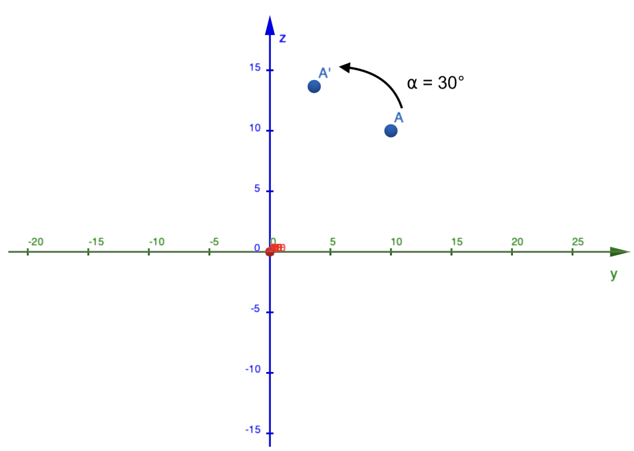
$$R_y(\alpha) = \begin{pmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{pmatrix}$$

$$R_z(\alpha) = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

1.2 Rotation a vector around the x-axis

Let's rotate a vector $A = (10, 10, 10)$ around the x-axis by 30°:

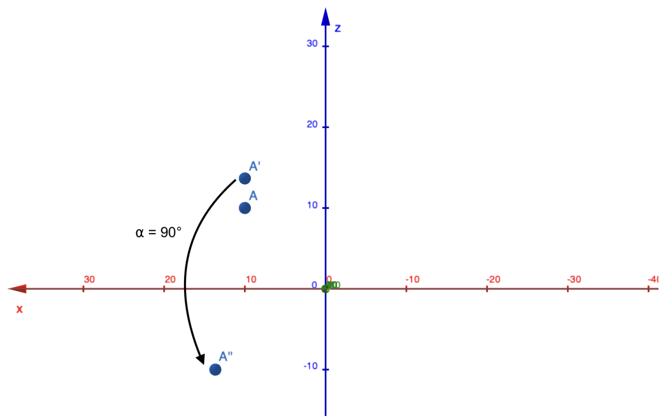
$$R_x(30) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(30) & -\sin(30) \\ 0 & \sin(30) & \cos(30) \end{pmatrix} \cdot \begin{pmatrix} 10 \\ 10 \\ 10 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.866025 & -0.5 \\ 0 & 0.5 & 0.866025 \end{pmatrix} \cdot \begin{pmatrix} 10 \\ 10 \\ 10 \end{pmatrix} = \begin{pmatrix} 10 \\ 3.660 \\ 13.660 \end{pmatrix}$$



1.3 Rotation a vector around the y-axis

Now, we will rotate that new vector $A' = (10, 3.660, 13.660)$ around the y-axis by 90°:

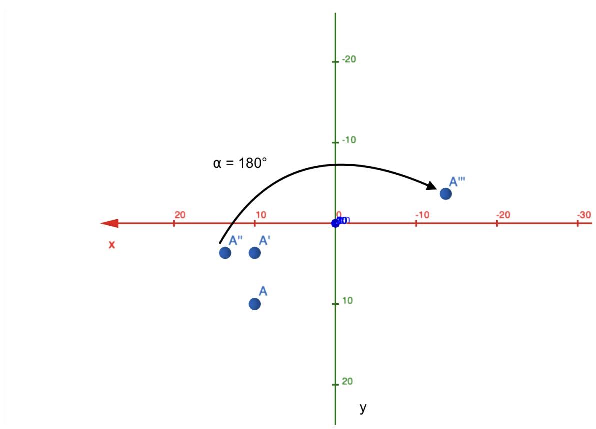
$$R_y(90) = \begin{pmatrix} \cos(90) & 0 & \sin(90) \\ 0 & 1 & 0 \\ -\sin(90) & 0 & \cos(90) \end{pmatrix} \cdot \begin{pmatrix} 10 \\ 3.660 \\ 13.660 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 10 \\ 3.660 \\ 13.660 \end{pmatrix} = \begin{pmatrix} 13.66 \\ 3.66 \\ -10 \end{pmatrix}$$



1.4 Rotation a vector around the z-axis

At last, we will rotate the vector $A'' = (13.66, 3.66, -10)$ around the z-axis by 180°:

$$R_z(180) = \begin{pmatrix} \cos(180) & -\sin(180) & 0 \\ \sin(180) & \cos(180) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 13.66 \\ 3.66 \\ -10 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 13.66 \\ 3.66 \\ -10 \end{pmatrix} = \begin{pmatrix} -13.66 \\ -3.66 \\ -10 \end{pmatrix}$$



1.5 Displaying the vectors

If the vectors need to be displayed e.g. on the xy-area you will leave the z-value out. Here's an example: we have the following vector given:

$$\begin{pmatrix} 14 \\ 10 \\ 10 \end{pmatrix}$$

Let's bring it into the form, to be displayed on the xy-area:

$$\begin{pmatrix} 14 \\ 10 \end{pmatrix}$$

Depending on the area, you want to display your vector, you need to leave the coordinate out, that does net appear in the name of that area e.g. in the xz-area, you need to leave the y-coordinate out etc.

1.6 Creating objects/structures out of the vectors

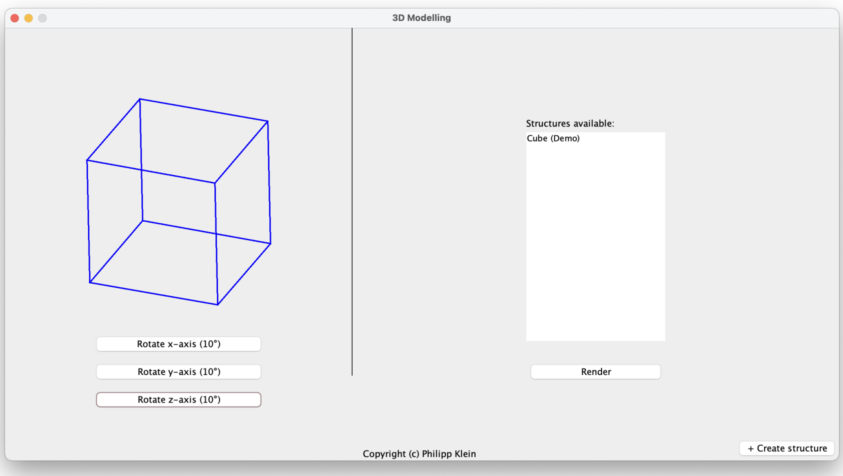
To now create objects or as I call them structures, we create a relation between those two vectors which essentially means, that there will be a line drawn between those two. Here's an example:

We want to create an triangle. The vectors are given as $\{(10, 0), (0, 10), (-10, 0)\}$ (They are being displayed on the xy-area). In order to form the triangle, we need to connect the following points: $\{\{(10, 0), (-10, 0)\}, \{(10, 0), (0, 10)\}, \{(-10, 0), (0, 10)\}\}$. Each connection is given as a tuple (pair of two vector).

2 How to use the software

2.1 Main screen

When you launch the software, you'll be on the main screen, where you can render and rotate your created structures. There is also a cube structure as a demo.

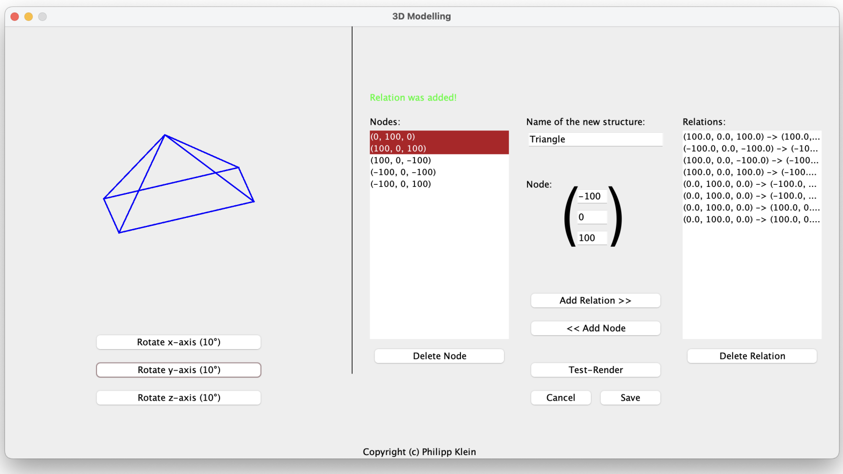


On the left side of the screen, you'll find the rendering area for the structure. You can rotate it using the buttons at the bottom. To create a new structure, click the '+ Create structure' button in the bottom right corner.

To select a structure for rendering, select one in the list and press the 'Render' button.

2.2 Creator screen

After pressing the '+ Create structure' button on the main screen, you'll find yourself on this screen:



On this screen, you have the following options:

1. **Naming Your Structure**: Upon entering the creator screen, you'll see a text field labeled 'Name of the new structure'. Here, you can give your structure a unique name.

2. **Adding Nodes**: In this software, I refer to vectors or points as 'Nodes'. To add a Node, locate the field inside the graphical representation of the vector/point. Here, you can type in the coordinates of the Node you wish to add. There will be placeholders for the coordinates to help you.

3. **Creating Relations**: To create a structure, you'll have to add relations between Nodes. Remember, a relation can only be made between two Nodes, otherwise an error will be given out. Once you've selected two Nodes, you can create a relation between them.

4. **Deleting Nodes and Relations**: If you need to remove a Node or a relation, you can do so by selecting them from their respective lists. Once selected, press the 'Delete' button located at the bottom of each list.

5. **Test Rendering and Rotation**: The software allows you to test render your structure. This means you can view your structure as it would appear when rendered. Additionally, you can rotate your structure around the x, y, and z-axis by 10 degrees, similar to the main screen.

3 References

[1] Rose, W. C. (2015). Mathematics and Signal Processing for Biomechanics. University of Delaware.
Online accessed: 02.08.2024. Available at: https://www1.udel.edu/biology/rosewc/kaap686/notes/matrices_rotations.pdf